CONTRIBUTIONS TO ADAPT THE TRIVALENT ELEMENTS OF ALGEBRA TO THE GRAPHIC-ANALYTICAL METHOD FOR DETERMINING THE HOURLY INDEX OF A THREE-PHASE TRANSFORMER

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REZUMAT. Articolul evidenţiază o variantă grafică a metodei alimentării în curentul continuu, aplicată în scopul identificării indicelui orar al unui transformator trifazat, variantă care are la bază o construcţie vectorială. În final, sunt prezentate contribuţiile autorilor la adaptarea variantei grafo-analitice a metodei alimentării în curent continuu la algebra elementelor trivalente.

Cuvinte cheie: transformator trifazat, versiunea grafo-analitică, algebra elementelor trivalente.

ABSTRACT. THE article shows a graphic version of DC method, applied in order to determine (to identify) a hourly index of a three-phase transformer, a version which is based on a vector construction. Finally, there are described the authors' contributions at the adapting with a graphic-analytical version of the DC method, to the algebra of trivalent elements.

Keywords: three-phase transformer, graphic-analytical version, algebra of trivalent elements.

1. INTRODUCTION

The using of the graphics or the graphic-analytical methods for studying the functioning of electrical transformers is known and applied in electrical engineering. In this context, we can mention: the phase diagrams for solving equations of the transformer operation, the graphical method and the graphic-analytical method applied for determining of the secondary voltage variation in load, the phasor diagrams used to identify the hourly index, when we know the wiring diagrams of the windings. The study confirms the possibility of finding a grafico-analytical solution for a version to the method of DC power.

2. THE GRAPHIC - ANALYTICAL VARIANT FOR THE DC POWER METHOD

In [1, 3] it is presented a graphic-analytical variant for the DC power method. The concept of a graphic-analytical variant refers to complex numbers [2]. A complex number can be represented in the complex plane (or plane of Gauss), as shown in Figure 1, where the notations have the following analytical meanings:

$$z = a + j \cdot b \tag{1}$$

$$j = \sqrt{-1} \tag{2}$$

$$a = r \cdot \cos\varphi \tag{3}$$

$$b = r \cdot \sin\varphi \tag{4}$$

$$r = \sqrt{a^2 + b^2} \tag{5}$$

$$\varphi = arctg\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right) = arctg\left(\frac{b}{a}\right) \tag{6}$$

The DC power method will give the nine analog signals that can be introduced into a table of code with three rows and three columns. The analog signals obtained from experimental tests may be replaced by digital signals and the analog code table is replaced by a numeric table, which contains signals denoted by (1), (-1) and (0).

Based on the information stored in the first line of the code matrix, determined experimentally, the authors have identified an analytical expression which gives the possibility to obtain by calculation, the hourly index searched. The three signals denoted by λ_{ab}, λ_{bc}, λ_{ca}, take, depending on the hourly index, one of the following values: (1), (-1) and (0).

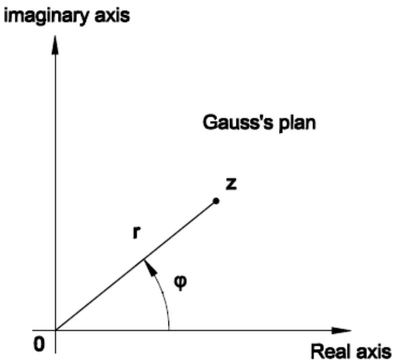

Fig. 1. The definition of a complex number.

The λ_{ab}, λ_{bc}, λ_{ca}, signals will be replaced by the complex numbers z_{ab}, z_{bc}, z_{ca}.

The expressions that define the complex numbers: z_{ab}, z_{bc}, z_{ca}, $-z_{ab}$, $-z_{bc}$, $-z_{ca}$, are established in relation to the Figure 2 and are shown in Table T1.

In the Gauss plane there are represented the complex numbers z_{ab}, z_{bc}, z_{ca}. It is performed the amount: $z = z_{ab} + z_{bc} + z_{ca}$ and it is calculated the sum argument:

$$\varphi = arctg\left[\frac{\mathrm{Im}(z_{ab} + z_{bc} + z_{ca})}{\mathrm{Re}(z_{ab} + z_{bc} + z_{ca})}\right] \quad (7)$$

The hourly index searched will result of the formula: $\mathbf{i^0} = \frac{\varphi}{30}$.

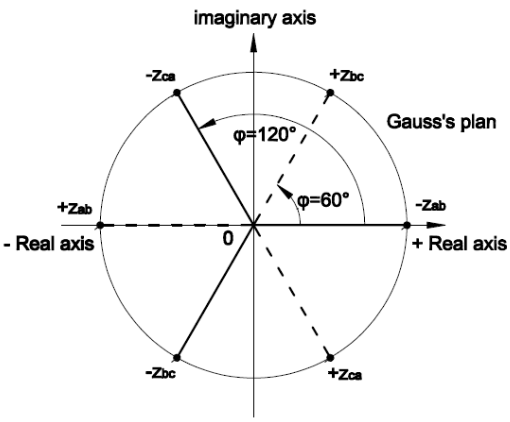

Fig. 2. The definition of the graphic-analytical version, for the method of DC supply.

Table 1

The result values of the experimental test		The complex numbers which are corresponding
λ_{ab}	1	$z_{ab} = -1 - j \cdot 0$
	-1	$-z_{ab} = 1 + j \cdot 0$
λ_{bc}	1	$z_{bc} = \frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}$
	-1	$-z_{bc} = -\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}$
λ_{ca}	1	$z_{ca} = \frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}$
	-1	$-z_{ca} = -\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}$

Finally, we get the formula:

$$\mathbf{i^0} = Q + \frac{1}{30} \cdot arctg\left[\frac{0{,}86 \cdot (\lambda_{bc} - \lambda_{ca})}{-\lambda_{ab} + 0{,}5 \cdot (\lambda_{bc} + \lambda_{ca})}\right] \quad (8)$$

where: Q – is a correction factor:

$$Q = 3 \cdot (3 + \lambda_{ab}) \quad (9)$$

The final values for the hourly index searched are set as follows:

$$\mathbf{i}_{\mathbf{final}} = \begin{cases} \mathbf{i^0} & \text{when } \mathbf{i^0} < 12; \\ \mathbf{i^0} - 12 & \text{when } \mathbf{i^0} > 12. \end{cases} \quad (10)$$

The verifications carried on the relationship (8), taking into account all the 12 possible cases, highlight two cases of doubt:

- the case of the hourly index **i** = 3, expressed by the matrix code: $\boldsymbol{G_3} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$;
- the case of the hourly index **i** = 9, expressed by the matrix code: $\boldsymbol{G_9} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$.

In the first case, λ_{ab}, λ_{bc}, λ_{ca}, we have the following values:

$\lambda_{ab} = 0$; $\lambda_{bc} = -1$; $\lambda_{ca} = 1$.

In the second case, λ_{ab}, λ_{bc}, λ_{ca}, we have the following values:

$\lambda_{ab} = 0$; $\lambda_{bc} = 1$; $\lambda_{ca} = -1$.

The situation of doubt is generated by the λab constant value, which for the hourly index **i** = 3 and for the hourly index **i** = 9, have the same value. Therefore, the relation (8) leads to the same result, for the $\boldsymbol{G_3}$ matrix and for the $\boldsymbol{G_9}$ matrix.

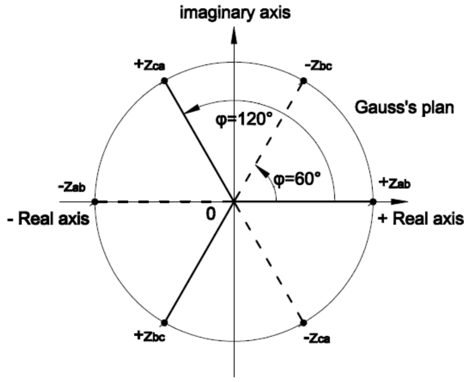

Fig. 3. The explanatory for the graphic-analytical version, after reversing of the position at the complex numbers.

In an attempt to resolve this situation of doubt, the authors have changed positions for the complex numbers $+z_{ab}$, $-z_{ca}$, $+z_{bc}$, $-z_{ab}$, $+z_{ca}$ şi $-z_{bc}$, in the complex plane, and have rotated them with the 180^0 in the clockwise. They obtained the variant which is presented in Fig. 3, which, without removing the previously reported cases of doubt, led to other expressions of complex numbers, presented in Table T2.

Table 2

The result values of the experimental test		The complex numbers which are corresponding
λ_{ab}	1	$z_{ab} = 1 + j \cdot 0$
	-1	$-z_{ab} = -1 - j \cdot 0$
λ_{bc}	1	$z_{bc} = -\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}$
	-1	$-z_{bc} = \frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}$
λ_{ca}	1	$z_{ca} = -\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}$
	-1	$-z_{ca} = \frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}$

The study results presented above, have highlighted the need to reconsider the expression for the correction factor. We sought that the expression of Q should be applicable in all 12 possible cases. In this case, it was proposed the following expression:

$$Q = 3\cdot(3+q) \tag{11}$$

where q is defined as:

$$q = \begin{cases} \lambda_{ab} & \text{when } \lambda_{ab} \neq 0 \\ \lambda_{ca} & \text{when } \lambda_{ab} = 0. \end{cases}$$

Therefore, (8), is replaced by the (12):

$$\mathbf{i^0} = 3\cdot(3+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca})}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca})}\right] \tag{12}$$

3. CONTRIBUTIONS TO ADAPT THE TRIVALENT ELEMENTS OF ALGEBRA TO THE GRAPHIC – ANALYTICAL METHOD FOR DETERMINING THE HOURLY INDEX

The authors have tried to adapt the graphic-analytical method, for the DC power method, to the algebra of trivalent elements. Considering a three-phase transformer, the algebra of trivalent elements means that the results obtained to determine the hourly index, with the DC power method, are presented in the form of nine electrical signals whose polarities are denoted as follows:

2 when the indicator deviates in the right;
1 when the indicator deviates in the left;
0 when the indicator remains immobile or deviates very little.

The expressions that define the complex numbers: z_{ab}, z_{bc}, z_{ca}, $2\cdot z_{ab}$, $2\cdot z_{bc}$, $2\cdot z_{ca}$ are shown in Table T3.

Finally, the authors have obtained formulas for identifying the twelve of hourly indices, which are presented below:

$$\mathbf{i_1} = 3\cdot(2+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}-2)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca})}\right]$$

$$\mathbf{i_2} = 3\cdot(2+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}-1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}-1)}\right]$$

$$\mathbf{i_3} = 3\cdot(2+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}-1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}-1)}\right]+1$$

$$\mathbf{i_4} = 3\cdot(1+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}-1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}+1)}\right]$$

$$\mathbf{i_5} = 3\cdot(1+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}+1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}+3)}\right]$$

$$\mathbf{i_6} = 3\cdot(1+q) + \frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca})}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}+2)}\right]$$

$$\mathbf{i}_7 = 3\cdot(1+q)+\frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}-1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}+3)}\right]$$

$$\mathbf{i}_8 = 3\cdot(1+q)+\frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}+1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}+1)}\right]$$

$$\mathbf{i}_9 = 3\cdot(1+q)+\frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}+1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}+1)}\right]+1$$

$$\mathbf{i}_{10} = 3\cdot(2+q)+\frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}+1)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}-1)}\right]$$

$$\mathbf{i}_{11} = 3\cdot(2+q)+\frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca}+2)}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca})}\right]$$

$$\mathbf{i}_{12} = 3\cdot(2+q)+\frac{1}{30}\cdot arctg\left[\frac{0{,}86\cdot(\lambda_{bc}-\lambda_{ca})}{-\lambda_{ab}+0{,}5\cdot(\lambda_{bc}+\lambda_{ca}-2)}\right]$$

Table 3

The result values of the experimental test		**The complex numbers which are corresponding**
λ_{ab}	2	$2\cdot z_{ab}=2+j\cdot 0$
	1	$z_{ab}=1+j\cdot 0$
λ_{bc}	2	$2\cdot z_{bc}=-1-j\cdot\sqrt{3}$
	1	$z_{bc}=-\frac{1}{2}-j\cdot\frac{\sqrt{3}}{2}$
λ_{ca}	2	$2\cdot z_{ca}=-1+j\sqrt{3}$
	1	$z_{ca}=-\frac{1}{2}+j\frac{\sqrt{3}}{2}$

4. CONCLUSIONS

✓ Considering a three-phase transformer, the results obtained in determining the hourly index with the DC power method will give the nine electrical signals, denoted by the trivalent elements of algebra, which can be inserted into a table with three lines of code and three columns. The signals obtained from experimental tests, noted with the trivalent elements of algebra, will be: (2), (1) and (0).

✓ Based on information stored in the first line of a matrix code, determined experimentally, the authors have identified the analytical expressions that give the possibility to obtain by calculation, the hourly index searched.

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